**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer: The service begins after 10 min from vehicle drop of which will now take more than usual time so new mean is 45 + 10 = 55

The probability that it will take more than 1 hour to complete.

So, the probability that service manager cannot meet his commitment is 0.2676 which is 26.76%

Solution is in python notebook.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans:

False.

The probability of employee age more than 44 is 15.86%

The probability of employee age between 38 and 44 is 34.13%

Solution is in python notebook.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: True

The training program for employee under the age of 30 at the center would be expected to attract about 36.48449 employees, which is approximately 36.

Solution is in python notebook.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer:

**For 2X1 :**

The random variable 2X1 is obtained by scaling X1 by a factor of 2. Since X1 follows a normal distribution, scaling it by a constant also result in a normal distribution.

**For mean:**

The mean of 2X1 is obtained by multiplying the mean of X1 by 2 therefore

µ2X1 = 2µ

**For variance:**

The variance of 2X1 is obtained by squaring the scaling factor and multiplying it by the variance of X1.

σ 22X1 = (22)σ 2 = 4σ 2

**For X1 + X2 :**

The random variable X1 and X2 is the sum of two i.i.d normal random variable. The sum of normal random variable is also a normal random variable.

**For mean:**

The mean of X1 +X2 is obtained by adding the mean of X1 and X2.

µX1 + X2 = µ + µ = 2µ

**For variance:**

The variance of X1 + X2 is obtained by adding the variance of X1 and X2.

σ 2X1 + X2 = σ 2 + σ 2 = 2σ 2

**Summary:** Both 2X1 and (X1 and X2) follow normal distribution, but with different parameters.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer:

D. 48.5, 151.5

Solution is in python notebook.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Rupee Range from 9.9008 to 98.0992 Crore rupees in Annual profit of the company 95% of the time.

Solution is in python notebook.

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: The fifth percentile of profit for the company is 16.9908 Crore rupees.

Solution is in python notebook.

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Division 1 has a larger probability of making a loss.

Solution is in python notebook.